Exo v1.0 Attention aux coquilles

Pour chacun des cas suivants, tracez la demande, la fonction d'élasticité, la recette totale RT et la recette marginale Rm:

a) $p = \dfrac{1}{\sqrt{q}}$

b) $p = \dfrac{1}{q^2}$

c) $p = 101 - q$

Solutions

a)

$$p = \frac{1}{\sqrt{q}} = \frac{1}{q^{1/2}} = q^{-1/2} \quad \Leftrightarrow \quad \left(q^{-1/2}\right)^{-2} = p^{-2} \quad \Rightarrow \quad q = \frac{1}{p^2} = p^{-2}$$

$$\varepsilon_p = \frac{dq}{dp}\frac{p}{q} = \left(-2p^{-3}\right)\frac{p}{p^{-2}} = -2 \cdot p^{-3} \cdot p \cdot p^2 = -2p^{(-3+1+2)} = -2$$

car

$$\frac{dq}{dp} = \frac{d(p^{-2})}{dp} = -2p^{-2-1} = -2p^{-3}$$

Demande

q	p
0.5	1.4142
1.0000	1.0000
2.0000	0.7071
3.0000	0.5774
4.0000	0.5000
5.0000	0.4472
6.0000	0.4082
7.0000	0.3780
8.0000	0.3536
9.0000	0.3333
10.0000	0.3162

$$RT = f(q) = p \cdot q = \frac{1}{\sqrt{q}} \cdot q = q^{-1/2} \cdot q = q^{1/2} = \sqrt{q}$$

$$Rm = \frac{dRT}{dq} = \frac{dq^{1/2}}{dq} = \frac{1}{2}q^{-1/2} = \frac{1}{2q^{1/2}} = \frac{1}{2\sqrt{q}}$$

b)

$$p = \frac{1}{q^2} = q^{-2} \quad \Leftrightarrow \quad q^2 = \frac{1}{p} \quad \Leftrightarrow \quad \left(q^2\right)^{1/2} = \left(\frac{1}{p}\right)^{1/2} = \left(\frac{1}{p^{1/2}}\right) \Rightarrow \quad q = \frac{1}{p^{1/2}} = p^{-1/2}$$

$$\varepsilon_p = \frac{dq}{dp}\frac{p}{q} = \left(-\frac{1}{2p^{(3/2)}}\right)\frac{p}{p^{-1/2}} = -\frac{1}{2} \cdot p^{-3/2} \cdot p \cdot p^{1/2} = -\frac{1}{2} \cdot p^{(-3/2+2/2+1/2)} = -\frac{1}{2}$$

car

$$\frac{dq}{dp} = \frac{d(p^{-1/2})}{dp} = -\frac{1}{2}p^{-(1/2)-1} = -\frac{1}{2}p^{-(1/2)-1} = -\frac{1}{2}p^{-(3/2)} = -\frac{1}{2p^{(3/2)}}$$

$$RT = f(q) = p \cdot q = \frac{1}{q^2} \cdot q = \frac{1}{q} = q^{-1}$$

$$Rm = \frac{dRT}{dq} = \frac{dq^{-1}}{dq} = -1q^{-2} = -\frac{1}{q^2}$$

À compléter…