MA(2) $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$ avec $\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$

Comme $E(y_t) = E(y_{t-j}) = 0$

$\gamma_j = E(y_t y_{t-j})$

Pour j=0

$$\gamma_0 = E(y_t y_t) = E\Big((1\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})(1\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E\Big(\varepsilon_t (\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_1 \varepsilon_{t-1}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_2 \varepsilon_{t-2}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E(\varepsilon_t \varepsilon_t) + E(\theta_1 \varepsilon_{t-1} \theta_1 \varepsilon_{t-1}) + E(\theta_2 \varepsilon_{t-2} \theta_2 \varepsilon_{t-2}) = \sigma_\varepsilon^2 (1 + \theta_1^2 + \theta_2^2)$$

Pour j=1

$$\gamma_1 = E(y_{t-1} y_t) = E\Big((1\varepsilon_{t-1} + \theta_1 \varepsilon_{t-2} + \theta_2 \varepsilon_{t-3})(1\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E\Big(\varepsilon_{t-1}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_1 \varepsilon_{t-2}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_2 \varepsilon_{t-3}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E(\varepsilon_{t-1} \theta_1 \varepsilon_{t-1}) + E(\theta_1 \varepsilon_{t-2} \theta_2 \varepsilon_{t-2}) = \sigma_\varepsilon^2 (\theta_1 + \theta_1 \theta_2)$$

Pour j=2

$$\gamma_2 = E(y_{t-2} y_t) = E\Big((1\varepsilon_{t-2} + \theta_1 \varepsilon_{t-3} + \theta_2 \varepsilon_{t-4})(1\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E\Big(\varepsilon_{t-2}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_1 \varepsilon_{t-3}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big) + E\Big(\theta_2 \varepsilon_{t-4}(\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2})\Big)$$
$$= E(\varepsilon_{t-2} \theta_2 \varepsilon_{t-2}) = \sigma_\varepsilon^2 (\theta_2)$$

Pour j>2

$\gamma_j = 0$

$$\rho_0 = \frac{\gamma_0}{\gamma_0} = 1$$

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\sigma_\varepsilon^2 (\theta_1 + \theta_1 \theta_2)}{\sigma_\varepsilon^2 (1 + \theta_1^2 + \theta_2^2)} = \frac{(\theta_1 + \theta_1 \theta_2)}{(1 + \theta_1^2 + \theta_2^2)}$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{\sigma_\varepsilon^2 (\theta_2)}{\sigma_\varepsilon^2 (1 + \theta_1^2 + \theta_2^2)} = \frac{\theta_2}{(1 + \theta_1^2 + \theta_2^2)}$$

Pour j>2

$\rho_j = 0$